

Mining Surveys and GIS Application

A Contemporary Introduction for Engineers and Geoscientists

A practice-oriented training module

Nashon Adero

Taita Taveta University (TTU), Voi, Kenya. www.ttu.ac.ke

nashon.adero@ttu.ac.ke | nashon.adero@gmail.com

Duration: 5 days (20 hours) | **Prerequisites:** College-level mathematics and basic computer skills | **Course fee:** 250 USD per head

Foreword

Mining Surveys and GIS Application: A Contemporary Introduction for Engineers and Geoscientists module has been prepared to address the urgent need in Africa for modern, practice-oriented and locally relevant training of surveyors and engineers to be engaged actively in surveying and mapping related to mining and geosciences. The contents have mainly drawn on the author's experience in Kenya as a tunnel surveyor and university lecturer of engineering surveying, mining surveying, and GIS. The material shared here reinforces the contents of a modern book on Geomatics, which is hereby recommended as a reference book for further topics with details on executing modern surveying and geomatics engineering assignments. The book is **Project Design for Geomatics Engineers and Surveyors, Second Edition (2023)** by Clement Ogaja, Nashon Adero, and Derrick Koome.

The author of the module, Nashon Adero, is a lecturer of Engineering Surveying, Mining Surveying, and GIS in the Department of Mining and Mineral Processing Engineering at Taita Taveta University, Voi, Kenya. He is a geospatial and systems modelling expert and a member of the System Dynamics Society, among many other professional affiliations. He has acquired multidisciplinary and multi-sector experience exceeding 15 years as a consultant and thought leader on emerging skills development issues and multicriteria decision-support models, a tunnel surveyor, a policy analyst on infrastructure and economic services, a youth mentor, a business development manager, and a university lecturer. He has authored, co-authored, and co-edited various academic and policy papers, books, and opinion pieces.

Foreword

TABLE OF CONTENTS

Foreword	2
Preface	5
Module Overview	7
Key Definitions	8
1. Purpose of the Module.....	10
1.1 Motivation	11
1.2 The Context.....	12
1.3 Training and Assessment.....	14
2. General Considerations	15
2.1 The Rich History and Evolution of Surveying	15
2.2 Fundamental Principles and Concepts	18
2.3 Common Instruments and Accessories.....	20
3. Underground Surveying.....	23
3.1 Practical Application of Underground Surveying.....	23
3.2 Transferring Survey Controls	24
3.3 Tunnel Surveys	25
4. Aspects of Setting Out and Deformation Monitoring.....	31
4.1 Slope Planning and Horizontal Alignment	31
4.2 Deformation Monitoring	33
5. Mining Applications.....	35

5.1	The Future of Mining Surveys	35
5.2	Classical and Emerging Features of Mining Surveys	36
5.3	Evolution in Instrumentation and Methods for Mining Surveys	37
5.4	Maps and GIS Application in Mining Surveys	42
5.4.1	Key types of maps used in mining surveys	43
5.4.2	GIS application in mining	45
5.5	Policy, Legal and Regulatory Aspects in Mining Surveys	47
6.	Bibliography	50
7.	Practice Exercises	51
7.1	GIS Application Exercise.....	52
7.2	Engineering Application Exercise	53

Preface

The ever-evolving stages of industrial revolution situate the mineral resource sector in a position of strategic social and economic significance. Why? The main reasons fall within the mainstream of the evidently mineral-driven future of our civilisation. It is acknowledged that earlier civilisations also relied on minerals, though to a lower extent in tandem with the lower population and standard of living of that period.

Today's rising living standards and global population drive a shift to higher consumption rates of minerals. Key examples can be seen in the increasing per capita consumption of iron ore and steel with increasing living standards, e.g., steel consumption of less than 1 tonne/capita in Africa against 5 tonnes/capita in China and more than 12 tonnes/capita in high-income countries, the increasing uptake of electric vehicles and high-end consumer electronics, the growing policy significance of pursuing the concepts of circular economy and green economy, and the need to decarbonise the material value chain in the face of climate change.

Interest in mining is growing beyond land into the sea and up to the space. With space technology gaining currency as a potent modifier of civilisation including the prospects for quantum information science, space/aerospace research is attracting scientific curiosity while calling for new materials adapted to the unique conditions in space. Minerals constitute the materials required for such extra-terrestrial missions.

Mining sector governance has consequently become a key area of concern. Applied precision science is critical to meeting the key policy objectives of sustainability, transparency, openness, stakeholder participation, compliance monitoring, fairness,

equity, safety, efficiency, productivity, among others. Mining surveys, which are fundamentally engineering surveys applied to address the planning, design, production, facility management and environmental monitoring needs in the mining industry and the sector in general, answer to the key demands of accuracy and precision in all these important aspects.

Young minds are adding a strong voice to the vision that the modern mining sector should align to. In my experience, young graduates of mining engineering from Taita Taveta University (TTU), Kenya's premier university of mining, have been sharing gratifying testimonies of the marketplace significance and rewards of surveying and GIS application. Since 2014, I have been teaching the BSc mining engineering students surveying and GIS units, spread out from the third to the fifth year. In their experience during industrial attachment and after graduating, surveying and GIS together make up a key employability skill.

Several reasons could be advanced for the recent lowering of the barriers to entry into the practice of mining surveys with related GIS application. First is the widespread availability of survey instruments and tools as well as Earth observation services that provide ready data and easily accessible imagery for GIS analysis. Second is the ubiquity of land-related rights and other human rights issues that demand spatial precision and spatially enabled models for effective resolution within the complex mining-environment-society nexus. Thus, a practice-oriented module that demystifies the key concepts, principles and applications of surveying and GIS in mining should be resourceful to all students and professionals actively engaged in the mining sector.

*Nashon J. Adero – Department of Mining and Mineral Processing Engineering (MMPE)
Taita Taveta University, Voi, Kenya*

Module Overview

Digital platforms for disseminating knowledge are the new normal in the post-pandemic education landscape. Practice-oriented modules depicting the evolving arc of knowledge from various disciplinary perspectives are resourceful in meeting the goals of a new era in a world recovering from the ravages of COVID-19.

A module on mining surveys addresses an urgent and important gap, being a segment that does not have many specialists yet technological innovations in surveying and interest in Africa's mining sector keep advancing. Designed to be an instructor-led module covering **five days (20 hours)**, this module is meant to be a concise yet rich and compelling introduction for engineers and geoscientists engaged in various professional ways in the mining sector. The module is designed to provide students, practitioners, and scholars with current insights into the application of surveying to mining while outlining the rich prospects and opportunities of geospatial technologies that transcend terrestrial applications to include offshore solutions. It contains exercises drawn from practical cases, which bring to life the otherwise abstract concepts and theories characterising advanced surveying.

The significant content of modern aspects of mining surveys makes this module an exciting training resource that will equip learners to confront challenging survey assignments in the mining sector innovatively. Upon completion, the learners will have developed competencies in providing an insightful reinterpretation of the prospects of surveying and mapping for the future of mining as a fabric of modern civilisation.

Key Definitions

Earth observation (EO): Refers to capturing data and information on the Earth and its component systems mainly through remote sensors (remote sensing), but also supplemented and enhanced by ground-based controls and surveys.

Geoid: An equipotential surface approximating the mean sea level.

Geo(spatial): Describes data, methods, and tools which are referred to, or associated with, specific locations in the real world.

Geoprocessing: Refers to the automated processing of GIS data using a suite of tools in GIS software for analysis, modelling, and management of the datasets to reveal new information.

Georeferenced: Specifically used to mean that the data and attributes in question are linked to a well-defined reference system of locating features on, below, or above the Earth's surface.

GIS: Geographic(al) Information System. To capture its broader and evolving meaning in this study, this module redefines GIS as an intelligent digital geospatial nervous system with adaptive means of delivering integrated solutions, mapped to scale to support transparent decisions within a democratised, inclusive and evolving engagement cycle.

GNSS: Global Navigation Satellite Systems – the collective term for all the satellite systems providing navigation and positioning services, namely, GPS, GLONASS, Galileo, Compass/Beidou, and others in the making.

Immersive technologies: Technologies which extend reality by integrating virtual contents and emulating the physical world, in this case specific to virtual reality (VR), augmented reality (AR), and mixed reality (MR).

Machine learning (ML): Considered a subset of artificial intelligence, machine learning uses computer algorithms to build data-driven models that improve automatically through self-learning to make predictions and decisions as guided by training data and experience, without any explicit programming to do so. Also considered as part of ML are: deep learning, probabilistic learning, reinforcement learning, transfer learning, decision trees, and

Key Definitions

genetic algorithms.

Map: The author defines a map as a selective abstraction of real objects over a given time period on a chosen surface to scale using a standard mathematical reference model of the Earth which, for the Earth's portion being represented, best preserves the key features of interest: size, shape, direction, or distance.

Mineral: Occurring in the form of ores from which they can be extracted at reasonable economic costs through the process of mining, “mineral” in this case is a naturally occurring product of geological processes which is inorganic, solid, crystalline with a fixed structure, and with a chemical composition which is either fixed or varying within certain defined limits.

Mining block: This is a spatial unit, essentially a cadastral unit, specified to be bound by a given number of arc-seconds of latitudes and longitudes (graticule). According to Kenya’s Mining Act of 2016, a mining block has the dimensions are 15” of latitude by 15” of longitude in the satellite geocentric WGS84 system.

Mining cadastre: This refers to the system of registering land-based interests in, and rights to, the ownership and operation of a mine. Through an online portal, the mining cadastre enhances online applications for legal permits and spatial verifications.

1. Purpose of the Module

This training module is designed to equip learners with fundamental knowledge in surveying and mapping as applied to meet the key data and accuracy requirements for sound design, integrated planning, spatial analysis and modelling, and similar geodata-related and multi-criteria decision-making in the mineral resources sector. The target group are senior college students, engineers, and natural resource managers whose principal study and research interests or occupational engagements are, or will be, in the mining sector. The module has been designed to cover **five days (20 hours)**.

The module learning outcomes (MLO) are hereby defined as the ideas, skills, and competencies the course participants should be able to articulate and implement theoretically or pragmatically after completing the module. By the end of this module, the participants should be able to deliver effectively on these MLOs:

MLO 1: Identify the main spatial data, spatial information, and modern survey instruments, accessories and tools needed to effectively plan, design, implement, maintain, and monitor various stages of mining given different spatial settings – terrestrial, offshore, airborne, or spaceborne.

MLO 2: Understand and articulate the significance, fundamental concepts, and applications of surveying and mapping techniques and technologies within the evolving global mining industry and the wider mining sector in general.

MLO 3: Apply the acquired knowledge to address social, environmental, economic as well as research and policy questions that need explicit spatial metrics to resolve through an integrated multicriteria approach, primarily using modern

Purpose of the Module

geographic information systems.

Since this module has key contents reflected in a book co-authored by the module author, the book is recommended as reference material for enhanced breadth and depth in surveying and mapping: [Project Design for Geomatics Engineers and Surveyors, Second Edition \(2023\)](#) by Clement Ogaja, Nashon Adero, and Derrick Koome.

1.1 Motivation

For many, the disruptive prospects of 5G and data transmission, driverless and flying cars, miniaturisation and uncrewed aerial systems, extended reality and the metaverse, the quantum revolution in computing, secure networks and cryptography, and big data may appear to be overly captivating, on the bleeding edge, and resting on the distant horizon at best. From confirmed cases by early adopters, these are cutting-edge innovations that will soon reach technological maturity and eventual saturation in the not-so-distant horizon. Space and time remain key elements in parametric decision-making, made possible using precise spatio-temporal parameters. As a result, there is growing interest in actionable location-based intelligence and similar proceeds and products of advanced geospatial technologies and innovations. including the spatial data generated within the big data ecosystems that support automation and various applications of artificial intelligence (AI) and machine learning.

The mining sector is not spared these disruptive changes and is also poised to be among the key beneficiaries of these developments. Mining as an enterprise requires business model innovation to thrive, hence well-informed de-risking mechanisms.

Sector-wide risk modelling relies on explicit spatial metrics to derive actionable location-based intelligence on the risk landscape. Based on the procedures and products of surveying and spatial models, solutions in support of long-term de-risking mechanisms with parametric policies can be achieved.

In the mining industry, technological innovation in in-situ instrumentation and remote data capture are driving a major transformation in the routine practices and labour participation models. New jobs, such as workflow programmers who can optimise human-to-machine matching for maximum outputs, are expected to rise. Understanding the fundamentals of mining surveys is, consequently, a key prerequisite for active engagement in transforming the mining sector outcomes.

1.2 The Context

Mining surveys deal with the accurate geospatial measurements needed to plan, design, implement, assess, and monitor mining activities including their environmental impacts while ensuring safe practice and post-mining environmental responsibilities. Large-scale mining activities involve the construction of access routes and structures on the surface and underground, all of which demand accuracy and precision in geolocation as well as vertical and horizontal alignments.

The principles of engineering surveys apply at the finer stages where stringent positional accuracy requirements are necessary. Engineering surveys find application in land-based engineering infrastructure and construction projects such as drainage channels, tunnels, roads, bridges, tall buildings, railways, airports, dams, among other human-made structures.

This module covers the key practical aspects of surveying and mapping in aid of

Purpose of the Module

planning, designing, setting out, constructing, maintaining, and safety monitoring of activities and structures in a mining environment as well as carrying out measurements for quantitative estimates. Imagery-based solutions from terrestrial, airborne and spaceborne technologies usually find applications in assessing and monitoring the mining environment and in 3D models and analytics that aid in quantitative estimates (e.g., estimating stockpile volumes from laser scanning, analysing classifying land use and land cover changes from aerial photography obtained using aircraft or drones or satellite imagery from space satellites dedicated to Earth Observation services, among others).

The future of the mining sector is attracting growing curiosity, especially in Africa as a young player in the sector and a rich reserve of at least 30% of the world's known hydrocarbon and mineral resources. In the mining sector, the Fourth Industrial Revolution or Industry 4.0 has informed new agenda on mining innovations under Mining 4.0. There are ready examples in automation, robotics, artificial intelligence, extended reality or immersive technologies, among others. Sustainable mining practices are expected to benefit from such advances.

Geodata or geospatial data and Earth Observation (EO) technologies are central to developing and scaling multicriteria spatial decision support systems to be responsive to the fast-changing demands of sustainable mining. Engineering surveys are critical to ensuring accuracy and precision in planning, designing, setting out, implementing, maintaining, and condition monitoring of the works associated with both surface and underground mining activities. Advances in terrestrial, offshore, airborne and spaceborne surveying and mapping technologies continue to enhance

safety, productivity, and efficiency in the mining sector including addressing land-related human rights and meeting environmental monitoring and management imperatives.

1.3 Training and Assessment

This practice-oriented module will be covered through interactive sessions as well as self-learning using uploaded guides. Problem-based learning, project-based examples, and group assignments will be used to improve the quality of delivery and engagement. Coursework will account for 50% of the assessment and a final written examination will account for the remaining 50%. The grading will range from **5.0** (less than 40% - fail) to **1.0** (more than 85% - excellent). The intermediate grades from 4.0 to 1.0 will vary by 0.3 and 0.7 as follows: 3.7, 3.3, 3.0, 2.7, 2.3, 2.0, 1.7, 1.3, and 1.0.

2. General Considerations

2.1 The Rich History and Evolution of Surveying

Historical backgrounds enable learners to gain insights into the subtle and sublime evolutionary stages of any disciplinary field, while simultaneously appreciating the amazing transdisciplinary megatrends defining modern times. As one of the oldest fields, Surveying is no exception. Surveying practice has evolved from the use of crude tools and chain and plane table surveying to modern spaceborne, airborne and sensor-based solutions generating big data on a daily basis.

Surveying and mining share a rich history that dates back to the ancient times. Egyptian surveyors subdivided the land by the Nile River three millennia ago. The Great Pyramid at Giza was built in 2700 BC. Land-based taxation purposes motivated developments in surveying towards a more accurate determination of land boundaries in Egypt around 1400 BC, laying part of the firm foundation for advancing the practice of cadastral surveys. The Greek advanced the science of geometry and invented the dioptra in the third century BCE as a precursor to the modern theodolite, 120 BC standing out for major improvements in Geometry thanks to the Greek. The Romans advanced the practice of surveying to the fields of surface mining and tunnelling, the Roman empire being the first civilisation to employ an official land surveyor.

Napoleon Bonaparte (1769 – 1821) was enthusiastic about accurate land surveying and precise maps and in this way made a key contribution to advancing the practice of surveying and mapping. Alexander von Humboldt, a German naturalist and explorer, further advanced the application of surveying and mapping to more abstract areas

including climate studies and the distribution of biota, laying the foundation for modern biogeography.

In Germany, historical mining activities influenced land surveying towards determining the land rights attached to mining blocks. Natively referred to as *Markscheidewesen*, mine surveying got established as a profession in Germany with the historical mining of silver in Saxony, which started in the early 12th century.

The Agrarian Revolution gave way to the advent of the 18 – 19th century Industrial Revolution in the UK, which led to the wide recognition of mine surveying as a profession. John Snow, in 1854, mapped out cholera outbreak in London and successfully linked the source to a contaminated water pump. The compelling need for more accurate surveys for boundaries and public works following these revolutions combined with expanded scales to cause a further development in the form of plane surveys and geodetic surveys, the latter applicable to large areas over which the curvature of the Earth must be part of the considerations during measurements and computations.

Advances in space technology, digital revolution, miniaturisation, and technology convergence are at the forefront in shaping the waveform and trajectory of modern megatrends in surveying and mapping. The geodata-driven digital revolution has radically redefined the profession and practice of surveying and mapping, introducing amazing prospects for geospatial careers in a future that will see a quantum leap in data transmission speeds and data processing technologies thanks to 5G, automation, artificial intelligence and machine learning, among others.

General Considerations

Recent advances in terrestrial, marine, airborne and spaceborne technologies for positioning, navigation and Earth observation have influenced unprecedented growth in geospatial data, essentially contributing to the “big data” revolution. Ready examples are found in modern laser scanners, radar, drones, mini-, micro- and nano-satellites, and Global Navigation Satellite Systems (GNSS).

The modern surveyor is a professional person with high academic qualifications and technical expertise and whose role keeps evolving with the data-driven digital revolution and fast-changing geoinformation technologies. The well-known Knowledge Doubling Curve also explains the blurring of disciplinary boundaries. It is, therefore, common nowadays to come across surveyors rightly rebranded as geospatial or geomatic(s) engineers. The International Federation of Surveyors (FIG) formalised the definition of a surveyor as adopted by the General Assembly on 23 May 2004 as follows:

A surveyor is a professional person with the academic qualifications and technical expertise to conduct one, or more, of the following activities:

- i. to determine, measure and represent land, three-dimensional objects, point-fields and trajectories;
- ii. to assemble and interpret land and geographically related information;
- iii. to use that information for the planning and efficient administration of the land, the sea and any structures thereon; and
- iv. to conduct research into the above practices and to develop them.

Surveyors are essential to the mining industry as they provide critical geospatial information for decision support involving all the other mining disciplines. Mining surveyors ensure the accurate measurement of areas and volumes mined as well as the representation of the surface and underground conditions on mining plans with the required spatial exactitude.

Engineering surveys are typically more stringent in accuracy requirements with tolerances for most linear measurements being a few millimetres, unlike assignments such as cadastral, boundary and topographic surveys where tolerances of several centimetres are allowed. The continuing evolution in industry dynamics has introduced other granular subdivisions of Geomatics, which apply the fundamental principles of engineering surveys: construction surveys, mining surveys, control surveys (both horizontal and vertical), topographic(al) surveys, detail surveys, route surveys, building surveys, hydrographic surveys, and so on.

Mining surveys, as part of applied precision science and engineering, utilise geospatial metrics to guide mining activities while making use of the principles of surveying, geodesy, mining, and geology. Practitioners may view mining surveying as a special category of applied engineering surveying, which is a geomatics discipline providing the knowledge and skills needed to conduct accurate geospatial measurements which are mainly used in construction projects.

Deformation monitoring to ascertain the performance and health conditions of completed physical infrastructure projects and related facilities is a key task executed through engineering surveying. The survey procedures help in ensuring that project planning and design, quality control, and quality assurance are strictly observed to ensure project implementation according to design standards and tolerances.

2.2 Fundamental Principles and Concepts

Engineering and/or mining surveys are critical to the operation and maintenance of engineering structures. In common practice, engineering surveys are applicable to land-based engineering infrastructure and construction projects, such as drainage

General Considerations

channels, tunnels, roads, bridges, tall buildings, railways, airports, dams, among other human-made structures. Due to the short distances involved in practice, the principles of plane surveying apply to both engineering and mining surveys, treating the Earth as flat. In some industry settings, very highly accurate surveys are necessary to meet the demanding standards of precision, an aspect of industrial metrology. This is a clear contrast to geodetic surveying, which involves extensive areas for which the curvature of the Earth must be considered.

Horizontal control through **traversing** and vertical control through **levelling** techniques are essential to carrying out engineering surveys. **Chainage** (e.g., 1 + 050 to mean 1.05 kilometres from the starting point) is used to give reference to a specific location from a given starting point when carrying out horizontal control. Horizontal alignment for infrastructure such as roads, railways, or tunnels requires horizontal control to achieve the design standards, guided by a network of known (reference) points. Vertical alignment is also necessary for engineering and mining projects because slopes or gradients must be set out to allow for accurate elevation differences, motion, and material flows within designed safety margins. Benchmarks (BM) and/or Temporary Benchmarks (TBM), which are points of known elevation, are set up to guide vertical control surveys.

During preliminary planning stages and initial excavations, the tolerances may be in the order of several centimetres, but they are improved to millimetres during final works, for example, when setting out **invert levels** for concrete lining in a drainage channel or a hydropower tunnel. It is standard practice to determine the most suitable instrument and the accuracy that any given engineering or mining surveying exercise

demands. The common instruments used for horizontal control are **optical solutions** (total stations, electronic theodolites), **mobile devices** with GNSS receivers (such as handheld GNSS or even smartphones), **geodetic solutions** (geodetic GNSS receivers), and **laser** precision instruments. For vertical control, automatic levels are the engineer's choice due to their ease of operation and high accuracy in determining **orthometric heights** (H), which are heights referred to the geoid as opposed to the less accurate ellipsoidal heights (h) obtained from satellite-based positioning using GNSS receivers.

Surveyors use checks to ascertain the accuracy of horizontal and vertical control. Having at least three known points to start with is recommended to ensure the redundancy required for a check. The checks can take the form of a mathematical formula, such as confirming if the sum of the interior angles measured for a polygon adds up to the expected value, or if a loop traverse returns a computed value that matches the known value at the starting point. If the error is small or relatively small, it is usually distributed proportionately among the occupied points, assuming a linear relationship. If the errors are gross and the instrument has been confirmed to be accurate through standard tests or calibration, then the survey exercise needs to be repeated.

2.3 Common Instruments and Accessories

The instruments and accessories shown in Fig. 2.1 are commonly used in engineering surveys. Curves and slopes are normally set out precisely using optical survey instruments.

General Considerations



FIGURE 2.1

Common instruments and accessories for engineering surveys. Photo credit: Nashon Adero, Taita Taveta University Survey Lab, 17-09-2022

Being precise and easy to use, automatic levels have become the industry standard for vertical control. Experience is key to setting up the instrument on a tripod (Fig. 2.2) and taking accurate readings on a vertically held levelling staff. Determining elevation differences (ΔH) and deducing the reduced levels of the points of interest are achieved using two standard methods: Height of Collimation Method and Rise and Fall Method. The former is the faster and preferred method.



FIGURE 2.2

Students learning to operate an automatic level. Photo credit: Nashon Adero (Taita Taveta University, Kenya).

Digital levels, which are more expensive and sophisticated, use barcodes to give digital records of elevation, a highly convenient feature for engineering surveys when higher speed, accuracy, and precision are desired in a project.

A **gyrotheodolite**, or surveying gyro, is particularly very useful in mining surveys. It comprises a gyroscope mounted to a theodolite and is the main instrument for orientation in mine surveying and tunnel engineering, where there is no clear view of the sky and hence GNSS cannot work. A gyrotheodolite is used to determine the

Underground Surveying

orientation of true north, except at or near (within 15 degrees of) the poles, where meridians converge, and the east-west component of the Earth's rotation is not to the extent that can help obtain reliable results. A gyrotheodolite is used to measure angles in both the vertical and horizontal planes. Its theodolite measures angles in the horizontal plane and its gyroscope measures angles in the vertical plane. The gyrotheodolite can measure the area, volume, and orientation of structures.

3. Underground Surveying

Underground surveying is a subsurface surveying technique that involves using optical surveying, magnetic surveys that produce subsurface images from measurements of the Earth's magnetic field, laser measurements, seismic surveys that use seismography to generate subsurface images from reflected shock waves, ground penetrating radar (GPR), measuring and mapping subsurface features, transfer of both horizontal and vertical controls from the surface to the underground environment, and alignment of underground structures, including mines, tunnels, and caves, among others.

3.1 Practical Application of Underground Surveying

Practical application areas of underground surveying include:

- i. **Vertical and horizontal alignment** of tunnelling works for bridges, subways, transportation tunnels, sewers, utility lines for power, fuels or communication, water tunnels, etc.
- ii. **Subsurface maps and models** to support exploration, engineering and construction of underground structures
- iii. **Locating mineral deposits**

- iv. **Locating geohazards:** sinkholes, landslides, etc.
- v. **Guiding the orientation** of shafting works and adits
- vi. **Mine safety applications** by monitoring stability/deformation and risk profiles including subsidence
- vii. **Estimating earthwork volumes**, including underbreaks and overbreaks
- viii. **Environmental application** in detecting underground contamination and monitoring remediation efforts
- ix. **Archaeological mapping** and reconstruction for various documentations of ancient sites
- x. **Studying subsurface geology** and mineral deposits as well as subsurface hydrology
- xi. **Petroleum exploration** for mapping subsurface reservoirs of oil and natural gas

3.2 Transferring Survey Controls

As a general rule, the surveyor should test the instruments to be used to ensure their accuracy, based on points of known (X, Y) coordinates and elevations (H). Optical solutions, GNSS, and lasers are commonly applied to transfer survey controls or reference points from one location to another. This step is necessary so that survey projects can be extended and executed within a controlled and location-specific framework of (X, Y) coordinates and elevations (H or Z). The tolerances vary, from relaxed decimetre or centimetre levels at the preliminary stage to stringent millimetre levels at the final construction stages. The process of increasing the density of control points or reference points in an area is also known as *densification*.

As a satellite-based solution, GNSS has made it easier to create (X, Y) control points without running long traverses from primary control points in a country, which used to

Underground Surveying

be the case before the invention and commercial availability of GNSS services. Static positioning techniques are commonly applied to densify control points for construction. Differential Real Time Kinematic (RTK) GNSS as well as software-based post-processing methods are also common, depending on the surveyor's choice.

Augmented GNSS services such as satellite-based augmentation system (SBAS) have helped to enhance the accuracy of GNSS positioning. Continuously Operating Reference Stations (CORS) are part of the solutions that have been advanced to increase the accuracy of GNSS positioning in many countries. There is a promising future of GNSS and its prospects for application in obstructed and confined environments, such as urban canyons, because of inventions such as multi-constellation satellite service capabilities and the renowned Z-Blade GNSS-centric technology.

Shafting and **plumblines** are a common means of vertical alignment for tunnelling works. **Benches** assist surveyors in transferring controls from the surface to underground, using a series of instrument stations. When using benches, traversing and levelling are the key procedures surveyors undertake to transfer horizontal and vertical control, respectively. Pegs are used for demarcating formation levels, marking them off with conspicuous spray paint. Other tools such as tape measure and strings are also useful and mostly applicable at the preliminary stages, for example, when guiding the course of excavation where a road should pass through.

3.3 Tunnel Surveys

Lasers are the main precision instruments used for alignment in tunnels. They have the advantage of generating a sharp, precise, visible line of reference under the poorly

lit underground environment characterising tunnelling and underground mining activities. Against this line of reference, surveyors can direct the vertical and horizontal alignments for excavations and estimates of quantities for “underbreaks” and “overbreaks”, as shown in the example of a tunnel being excavated (Fig. 2.3).

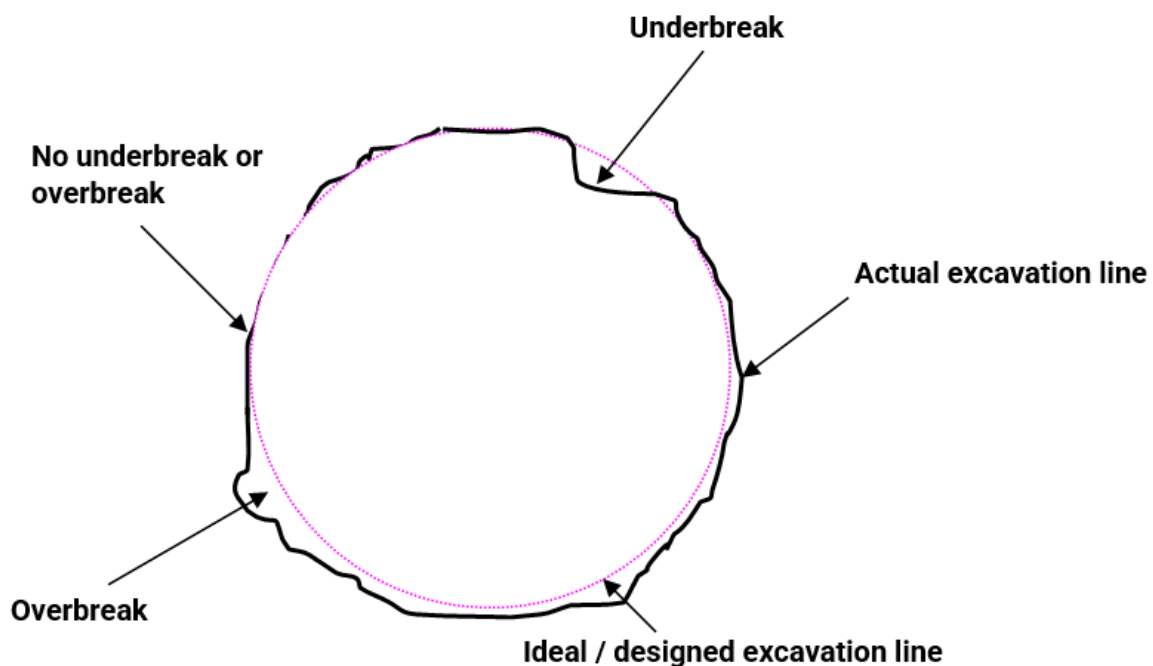


FIGURE 2.3

Cross-section of an excavated tunnel showing overbreaks and underbreaks.

A case study on tunnelling has been selected to exemplify the rigour and granularity of engineering and mining surveys. Long tunnels may require teams to work from both ends and meet up midway, hence the stringent accuracy requirements to avoid gross errors or minimise the systematic errors that can result in the teams veering off horizontally and/or vertically - never to meet up at all with massive losses of investment. Surveyors must consider the differences in coordinate and reference systems for transboundary tunnels to ensure that mathematical calculations and

Underground Surveying

error-minimisation techniques are harmonised.

Tunnels may serve the needs of mining, railway transport, road transport, non-motorised transport, water transport, hydropower production, among others. Notable tunnels and amazing products of engineering surveys in the world include, for example:

- i. The Delaware Aqueduct in the New York City water supply system (built 1939-1945) – the world's longest tunnel at 13.5 feet (4.1 m) wide and 85 miles (137 km) long.
- ii. Gotthard Base Tunnel from Zurich to Milan – the world's longest rail tunnel (57.09 km) as of June 2016 (built from 1996), at the cost of US\$10.3 billion.
- iii. Seikan Tunnel connecting the Honshu and Hokkaido islands in Japan (53.9 km, 23.3 km under seabed), completed in 1988, at the cost of US\$3.6 billion.
- iv. Channel Tunnel (50.5 km, 37.9 km under the sea), constructed from 1987-1994 at the
- v. cost of £4.65 billion.

Due to their execution in restricted underground environments with neither natural lighting nor physical landmarks for approximate positioning, tunnel surveys qualify as one of the strictest types of engineering surveys. Air must be pumped along long tunnels for the safety of workers, who should be in full safety gear. Underground water is a common challenge in tunnelling, hence the use of electric power to pump water out of the tunnels to facilitate survey procedures, such as fixing reference points along the designed centreline and temporary benchmarks, usually using concrete nails. Measures must be taken to avoid cases of electrocution due to faulty wires that may be exposed to the water. Blasting of rocks for easier tunnel excavation is common practice. For safety, geological investigations are required to determine rock type,

structure and composition and hence the right reinforcement needed, which may vary from shotcrete to iron bars or iron plates.

Surveying plays a vital role in the management of coastal zones, mineral exploration, and tourism. In coastal zone management, surveying is used to monitor the changing coastline, to identify potential hazards and to help plan and develop coastal areas. In mineral exploration, surveying is used to map geological features, to identify potential mineral resources, and to determine the best locations for mining operations. In tourism, surveying is used to create maps of attractions, to locate areas of natural beauty, and to plan and develop tourist sites.

An example of how laser beams are used to guide the excavation of a straight section of a tunnel with an excavation radius of 2.2 metres is shown (Fig. 2.4). This example is drawn from a practical application case of constructing a headrace tunnel for hydropower production in Kenya, the Sondu-Miriu Hydropower Project, for which the final radius after lining with concrete was designed to be 2.1 metres. More radial allowance is necessary during excavation to give room for sufficient thickness of concrete lining and for the movement of people, vehicles, machinery, and tools such as formwork of shutters and reinforcing bars.

Using traversing and levelling, targets are surveyed to be positioned at the correct (X,Y) locations and elevations (H) such that the laser beams are parallel and also tracing the correct slope according to the designed downstream/upstream slope of the tunnel section, **1 in 1000** in this practical case. The second pair of targets coinciding with the chainage of H' provides a necessary check. Measurements from the reference laser beam, both diametrically and vertically, aid in setting out the

Underground Surveying

measurements needed not only for accurate excavation and construction so as to replicate the designed properties, but also for volumetric estimates of earthworks. It can be appreciated from this example that both horizontal control and vertical control are critical and must be accurate to achieve such an amazing engineering feat.

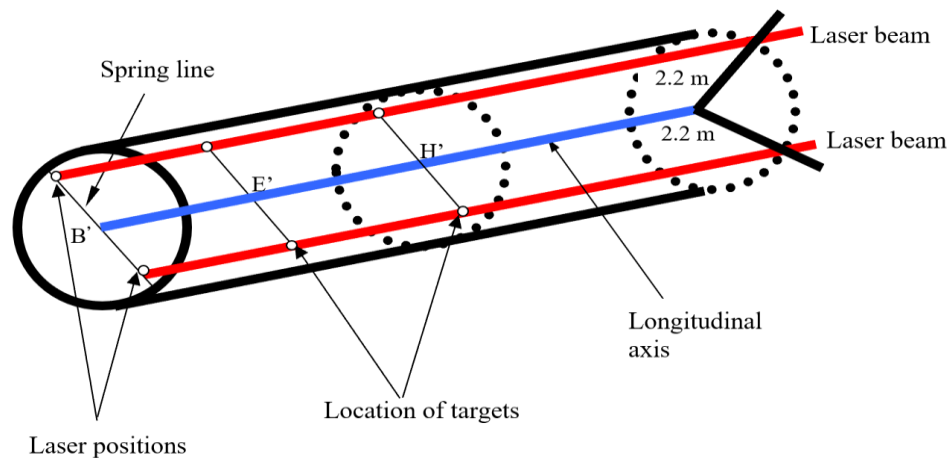


FIGURE 2.4

Straight section of Sondu-Miriu Hydropower Tunnel, Kenya. Drawn on site from surveying experience during civil works. Credit: Nashon Adero.

A laser beam was used to set out the curved section of the tunnel (Fig. 2.5). The exercise becomes more demanding because the designed curve geometry must be taken into account. As shown, the curve radius, R , is 200 metres, using the centreline as the reference as per the standard practice of setting out curves. The surveyor's challenge is to determine the offset AY , which gives the measurement from the laser beam to the point on the designed centreline. The surveyor needs to know the designed bearing of the straight/tangent to the curve at the point EC (known coordinates and chainage) and the chainage of A . As will be shown in the section for exercises, the mathematical equations for a straight line in rectangular coordinates

(plane surveying), curve length, and chord length are routinely applied to arrive at the solution for the offset AY. These underground application examples suffice for underground mining surveys as well since tunnelling is a necessary undertaking in such projects.

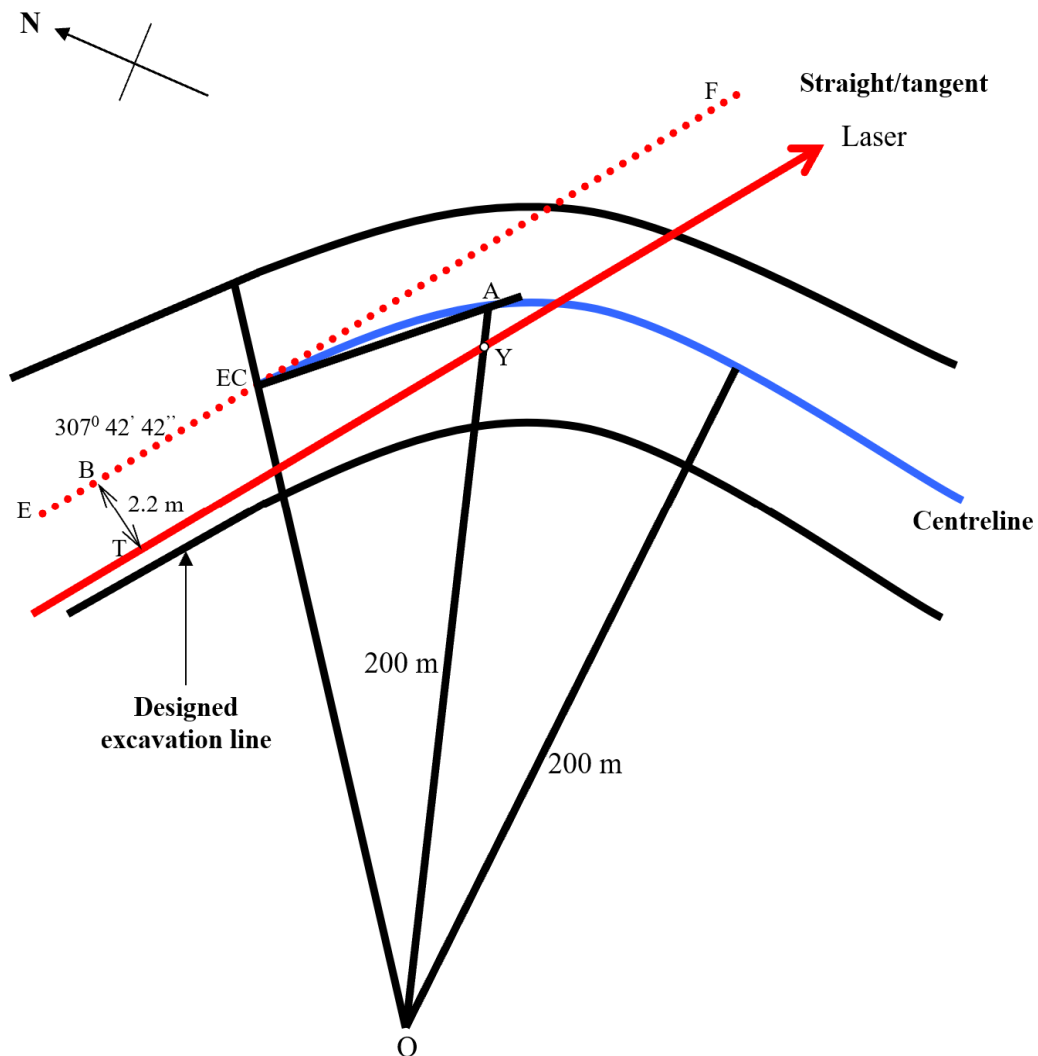


FIGURE 2.5

Curved section of Sondu-Miriu Hydropower Tunnel, Kenya. Drawn on site from surveying experience during civil works near the intake weir. Credit: Nashon Adero.

4. Aspects of Setting Out and Deformation Monitoring

Setting out of engineering structures relies on a network of control points and **offsets** calculated in reference to specific points on reference lines. Offsets are linear distance measurements perpendicular to the reference line at a point. The designed centreline of a structure such as a road or a tunnel is used as the reference line by default. Vertical control is also necessary to set out the right elevations and slopes.

4.1 Slope Planning and Horizontal Alignment

On design documents, slope may be expressed as **1 in X**, **1/X**, or in **degrees** (\emptyset). In any case, $\text{slope} = \tan(\emptyset) = dy/dx$ (vertical difference divided by horizontal equivalent), which is 1/X in this example, also equal to the tangent of the slope angle. The slope can also be expressed as a percentage, which is the same as $100 \tan(\emptyset)$.

Transport and logistics for mining activities rely on infrastructure, which may not be existing in remote areas and have to be surveyed and constructed. Railways demand a very gentle slope, normally less than a degree, because of the low friction between the steel wheel and the steel rail. Roads can manage a higher gradient or grade. A surveyor must take such factors into account during setting out.

A typical underground project that strictly requires the application of accurate surveying techniques is the tunnel section shown, extracted from the case of Sondu-Miriu Hydropower Project, Kenya (Fig. 4.1). It should be noted that this is a clean and renewable energy project, hence its relevance to greening operations in the normally energy-intensive large-scale mining industry.

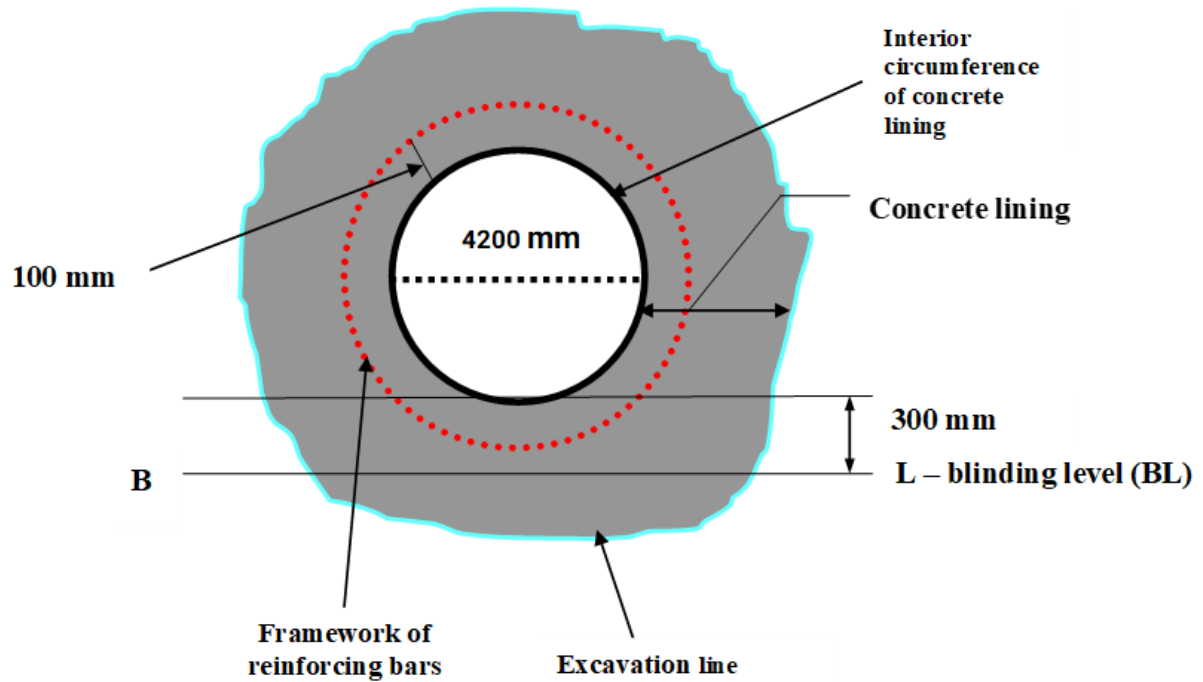


FIGURE 4.1

Reinforced concrete lining of a tunnel section with a design diameter of 4200 mm.

Horizontal and vertical alignment of the reinforcing iron bars inside the tunnel in Fig. 4.1 were accomplished using a total station and a level. Surveyors aligned an adjustable 15-metre-long formwork made of steel to assume the designed shape and orientation of the tunnel, after which the iron bars were laid on it to take on the shape and orientation of the formwork before the formwork could be collapsed and shifted to a new position. The aim was to end up with a tunnel of a circular shape once lined with reinforced concrete, as shown in Fig. 4.2.



FIGURE 4.2

Reinforced concrete lining of a tunnel section using optical solutions - a total station and a level. Photo credit: Nashon Adero.

4.2 Deformation Monitoring

Deformation monitoring involves carrying out systematic, precise, and periodic measurements on a structure in order to detect (usually small and gradual) displacements as a function of time in the structure away from the originally established positions, shape or geometrical relationships – normally due to induced stresses. Deformation monitoring is key to preventive maintenance. With technological advances, big data, Machine Learning and Artificial Intelligence, **predictive maintenance** is becoming a reality as well.

Examples of structures to be monitored include bridges, dams, buildings, tunnels, road and rail infrastructure, fuel storage tanks, and pipelines. Precise survey measurements help detect lateral and/or vertical shifts/movements/subsidence. The

measurement techniques employed can be as simple as measurements of radials in a tunnel using a levelling staff, such as within an excavated tunnel, to complex ones yielding sub-centimetre (millimetre) measurements using precision instruments and photogrammetry techniques. The survey methods applied to monitor structural deformations make use of traversing, levelling, laser surveys, photogrammetry/drone- and LiDAR-assisted generation of **point clouds**, or satellite-based positioning and imaging methods (GNSS/GPS/satellite images).

In summary, a surveyor needs to consider the following instrumentation and procedures for a complete deformation monitoring schedule:

- i. Instrumentation: Optical instruments, GPS/GNSS, imagery/photogrammetric set ups
- ii. Reference points are accurately determined
- iii. Monitoring points are placed on the structure with receptors, e.g., calibrated reflectors
- iv. Monumenting of the control points as a cautionary measure against the displacement of reference points
- v. Taking periodic measurements from the control points, used as reference stations, and taking readings to the monitoring points on the structure to obtain data on any changing geometries
- vi. Deformation analysis based on well-grounded theory to reflect on the ideal condition, against the observed conditions at different epochs/times
- vii. Reporting of the results and recommendations for necessary preventive interventions

5. Mining Applications

5.1 *The Future of Mining Surveys*

Minerals are driving the future of industrialisation and human civilisation. Battery and fuel technologies, for example, are gaining policy relevance in a world experiencing a growing uptake of sophisticated consumer electronics and electric vehicles. Lithium, nickel, cobalt, copper, aluminium, manganese, tantalum, platinum, palladium, and rare earth minerals are among the minerals considered critical to this future. Renewable energy integration into the mining sector is gaining policy momentum and relevance as an effective way of achieving decarbonisation targets, hence importance of surveying and mapping for geothermal, hydropower, solar, wind, and wave-energy projects.

The global mining industry is categorised as a heavy industry due to its energy-intensive demands for large-scale operations. McKinsey & Company estimated that mining was contributing 4-7% of the global greenhouse gas (GHG) emissions by 2020. Environmental responsibility in the mining sector is a key global issue in the pursuit of the UN Sustainable Development Goals (especially Goal 7, Goal 13, and Goal 15). Decarbonisation and meeting net-zero emission targets in the face of climate change are compelling policy goals. Green steel production using hydrogen is a key example of the attempts at greening the mining industry.

Informed by recent global developments, mining scholars have rightfully argued that, in terms of importance to society, mining should be accorded a weight that is no less than the weight of farming, fisheries, and forestry. Mining surveys, as a result, are

gaining prominence. Issues of job losses aside, automation is gaining importance in the global mining industry since it is key to safety enhancement.

Automation of operations in the mining sector including machine guidance requires accurate geospatial positioning techniques. Research on the bleeding edge also promises deep-sea mining as the next frontier of mining, making mining surveys even more critical in future and a key technical aspect of the blue economy. Extended reality or immersive technologies, such as virtual reality (VR), augmented reality (AR) and mixed reality (MR), are among the new avenues that geodata-driven technologies have opened up for the mining industry to reap immense benefits of enhanced productivity, safety, and cost-effectiveness.

5.2 Classical and Emerging Features of Mining Surveys

Mining surveys involve applied precision science and engineering that generates the geospatial metrics needed to guide mineral exploration and mining activities while utilising the principles of surveying, geodesy, mining, and geology. Mining surveys may be considered as a special subclass of engineering surveys applied in mining environments.

The life cycle of any mining enterprise, from exploration to post-closure activities, presents many phases in which the role of surveying and geospatial techniques and technologies remains critical. Spatial metrics are key to policy decision support, safety monitoring, mineral resource exploration, mine planning, mine design, determining land-related mining rights with exactitude, quantitative estimates of earthworks or materials on site, and post-mining rehabilitation or landscape restoration.

Mining Applications

Deposit processing involves the exploration, evaluation, and determination of the reserves of a deposit and supports a mining plan in regard to optimal use of the reserves. **Approval procedures** are a precondition when exploring and extracting mineral resources. They are the basis for the acquisition of a mining permit or license and access to the land where mining takes place.

Traversing and levelling are classical methods that continue to find application in mining fields, for horizontal and vertical alignment of both surface and underground mining activities. Theodolites, total stations, and levels are commonly used, complemented by laser precision instruments.

5.3 Evolution in Instrumentation and Methods for Mining Surveys

Mining surveys require adequate instrumentation to effectively address both surface and underground assignments. A *gyrotheodolite* is used for orientation in underground environments. Laser surveys (including airborne laser scanning) and the common optical solutions in engineering surveys are applied to deliver mining-related solutions as well.

Slope planning, pegging, setting out, deformation monitoring, and determining rights and legal liability as per mining blocks are common survey assignments in the mining sector. Since land and mining are strongly interconnected, cadastral surveys are critical to determining mining rights on land. Countries are advancing towards digital 3D cadastre so that mining rights can be better managed, including the effects of underground mining on the structures owned by neighbouring communities.

Airborne solutions continue to transform mining operations and exploration. **Airborne geophysical surveys** produce geophysical maps and are commonly conducted to map out the mineral potential of a territory. Control points are established to ensure that the final map can be georeferenced, hence enriching the final products with actionable location-based intelligence. Aerial photogrammetry combined with LiDAR aboard UAVs has been highly useful as a safer means of pit mapping and measuring the volume of stockpiles while conducting 3D modelling and monitoring. The photogrammetrically generated data is usually processed using automated software-assisted processes to create digital terrain models (DTM), digital elevation models (DEM), imagery that is both orthorectified and georeferenced, and topographic maps.

Spaceborne solutions rely on the global coverage of geolocation information (GNSS) and imagery data from space satellites for safe and cost-effective results for the mining industry. Mining and geological applications benefit from the data obtained from satellite image processing, georeferencing, orthorectification, feature extraction, and mosaicking.

Material detection is made possible using the short-wave infrared (SWIR) wavelength bands, obtainable from SPOT 4 and SPOT 5 satellite remote sensing as well as DigitalGlobe's high-resolution SWIR imagery. A **borehole camera** is part of the highly useful equipment in mining surveys. The 3D spatial coordinates of borehole data are analysed using GIS techniques and artificial neural networks (ANN) to reveal characteristics that are key to detecting geohazards.

Mining Applications

A borehole camera is used generally for geological mapping, geotechnical evaluation, structural analysis, orebody delineation, and hydrogeological assessment, with specific key functions as follows:

- i. ascertaining groundwater conditions and hydrogeological characterisation in a mining area – detects water inflows, aquifers, and permeable zones; shows location, colour, consistency, and the amount of precipitates. These key applications aid in environmental impact assessment, designing dewatering systems, and water resource management
- ii. viewing subsurface conditions for geological information on rock types, faults, fractures – hence supporting fracture logging by direct observations and measurement or using imaging functions and acoustic televiewers
- iii. viewing bedding planes, and recording the stratigraphy and lithology of the accessed units
- iv. documenting cracks or holes and leaking joints
- v. observing hole offsets and blockages
- vi. assessing rock quality and age – geotechnical investigations
- vii. provides images for identifying mineralisation zones and estimating the size and grade of the ore body, which supports mine planning

Remote sensing and GIS are together potent tools for assessing, quantifying, and monitoring the effects of mining on land and environment. They are important sources of the data and information that is increasingly required for sound decision support and policy development for sustainable mining practices in the broader mining-environment-society nexus. Advances in the resolution of satellite imagery are delivering more resourceful data for land use and land cover assessments across

mining areas. Initiatives such as Digital Earth Africa and Digital Earth Australia are providing analysis-ready data (ARD) and decision-ready data (DRD) from processed satellite imagery, making it even easier to generate spatial models that support decisions and policies on sustainable mining and reclamation of closed mining sites. Sentinel (10 m), PlanetScope (3 m), Pleiades (0.5 m), WorldView-3 (0.3 m), and hyperspectral images of sub-metre spatial resolutions demonstrate the innovations pushing the boundaries of satellite-based imagery solutions for the mining sector.

On mining sites are key structures such as storage tanks and dams, which do need regular deformation monitoring to ensure safety. Blasting and other highly vibratory activities on mining sites due to heavy machinery make subsidence monitoring a crucial undertaking. Land subsidence is a key safety risk that should be monitored keenly on mining sites. **Interferometry** is a technique whereby multiple repeat satellite radar images taken over a scene are integrated to detect small shifts, such as small ground movements that could well be in the order of a millimetre per year. The European Union's Copernicus Sentinel-1 satellites, for example, have been providing this important service that can generate industry-optimised analytics for actionable insights in aid of monitoring changes in landscapes affected by mining and quarrying activities. *Differential radar interferometry* or *Differential Interferometric Synthetic Aperture Radar (DInSAR)* has been a powerful means of monitoring small changes that lead to land subsidence in mining areas. This method, based on **active remote sensing**, is more effective and economical over large areas than the less effective point-by-point measurements obtained using classical optical solutions.

Mining Applications

Airborne Ground Penetrating Radar (GPR) facilitates remote data collection in hard-to-reach and hazardous areas, typical of most mining areas. These radar-based technological advances are a great boost to **subsidence engineering**. **Augmented GNSS** delivers centimetre-level accuracy for automation and machine guidance in mining. This solution is also applicable to route planning and fleet management, which are key to enhancing productivity in busy mining operations.

With offshore prospects in deep-sea exploration for minerals, surveys of the seafloor morphology or bathymetry make use of **multibeam acoustic and airborne laser systems**, nautical charts, and echo sounders for depth measurements. The use of **green laser** is transforming outcomes in airborne laser bathymetry (ALB). GNSS still finds application in navigation for safety and for transport of cargo during such exploration missions.

Technological innovation is enhancing offshore mineral exploration prospects using **autonomous underwater vehicles**. For sea-floor exploration, navigation and sampling use **autonomous robotic systems** that can work under high pressure, low temperature, and total darkness. Intelligent response to sound frequencies that trigger the dropping of weights used during immersion enables the return of the vehicles to the surface, relying on their buoyancy. Several case studies of these efforts exist with the GEOMAR Helmholtz Centre for Ocean Research Kiel, Germany, acting as a key example in deep-sea exploration for minerals.

Software developments continue to boost model development for planning and design related to mining. Block models can nowadays be readily generated using specialised commercial software, such as Surpac and MineSight 3D. Automated

cartographic solutions for mapping and planning at scale add to the long array of benefits of software development.

5.4 Maps and GIS Application in Mining Surveys

For a comprehensive description, map is redefined in this module as a selective abstraction of real objects over a given time period on a chosen surface to scale using a standard mathematical reference model of the Earth which, for the Earth's portion being represented, best preserves the key features of interest: size, shape, direction, or distance.

The emergence of automated cartography and Geographic Information Systems (GIS) has significantly enhanced and streamlined the process of creating digital maps. Despite the ongoing geospatial data-driven digital revolution, adherence to cartographic standards remains crucial and indispensable. Maps designed to inform mining decisions must comply with the fundamental standards, including:

- i. **Title** – It is advisable to concisely convey the overarching message of a map through an appropriate title.
- ii. **Scale** – Consistently indicate the map scale, ideally in the form of a scale bar, to maintain proportionality across varying display and print sizes.
- iii. **Coordinate system** – Provide information about the coordinate referencing system employed, whether geographic (spherical coordinates) or projected (linear/Cartesian or rectangular coordinates – more amenable to Euclidean geometry, algebra, and calculus).
- iv. **Map projection** – When the Earth's curved surface is projected onto a plane, cylinder, or cone, specify the map projection used, such as the Universal Transverse Mercator (UTM) and its corresponding zone.

Mining Applications

- v. **Orientation** – Indicate the North direction by selecting a suitable North arrow from the GIS software's gallery.
- vi. **Legend** – This crucial component serves as a guide to the map's elements, represented by various lines, symbols, and shades, among other features.

5.4.1 Key types of maps used in mining surveys

Geological maps and **geophysical maps** are two main types of maps used to study subsurface features. Geological maps and geophysical maps both provide essential information about the Earth's subsurface, but they differ in their objectives, data sources, and presentation.

Geological maps are used for a variety of applications in mining and natural resource management. While geological maps provide information about the composition, structure, and age of rock and sediment layers, geophysical maps are used to measure and map variations in the physical properties of rocks and sediment layers. Geological maps provide information about the location and type of ore and mineral deposits, geological structures, and fault lines. Geological maps are also used for geological hazard mapping and for planning underground construction projects.

Geophysical maps are used to identify subsurface features and to infer the composition and structure of the subsurface. They are used in the oil and gas industry to identify potential drilling sites and in mineral exploration to identify potential ore and mineral deposits. Geophysical maps are also used in environmental studies to identify possible sources of groundwater contamination and in engineering studies to identify potential sites for underground construction projects.

In summary, here is a detailed tabular comparison between the two types of maps.

Aspect	Geological Maps	Geophysical Maps
Purpose	Display rock types, structures, geological history, and identify resources, hazards, and features	Investigate subsurface properties and structures, identify mineral, energy and groundwater resources, and aid in environmental assessment and hazard mitigation
Data Sources	Field observations, rock sampling, drilling, laboratory analysis (texture, mineralogy, age), aerial photography	Airborne or ground-based surveys (magnetometry, gravity, seismic, electromagnetic induction, ground-penetrating radar)
Representation	Colors, symbols, lines, and annotations	Contours, color gradients, or grid values
Scale	Various scales, from outcrop to basin or regional scales	Various scales, from regional to national or global scales
Types	Bedrock, surficial, structural, stratigraphic, tectonic, thematic maps, cross-sections, diagrams	Magnetic, gravity, seismic, electromagnetic, radar maps, and other physical parameters (resistivity, conductivity, density)
Key Features	Rock units, contacts, faults, folds, stratigraphy	Anomalies, interfaces, structures, isopachs, isochrons
Interpretation	Geological knowledge, field experience, lab data analysis: Requires interpretation of field and laboratory data to infer the geologic history and processes	Geophysical knowledge, data processing techniques, and integration of multiple datasets: Requires integration and modelling of multiple datasets to infer subsurface features

Mining Applications

	that have shaped the landscape, which can be affected by factors such as age, deformation, erosion, and deposition	and properties, which can be affected by factors such as depth, resolution, noise, and geologic complexity
Applications	Mineral exploration, civil engineering, land use planning, groundwater resources, education, research	Oil and gas exploration, mineral exploration, earthquake studies, archaeology, groundwater management, geohazard mitigation
Primary Users	Geologists, engineers, urban planners	Geophysicists, geologists, engineers

5.4.2 GIS application in mining

A GIS application is an automated process that generates a spatially oriented product or result needed by a user. GIS applications may include map update or map production, data query and display, spatial analysis, or other processes that use GIS software and geographic data (Ogaja, Adero, & Koome, 2023, p. 149).

There is a significant role of GIS in the mining industry, through offering valuable insights and tools for informed decision-making, resource management, and environmental protection. GIS technology is central to multicriteria analysis, hence to the implementation of modern multicriteria spatial decision support systems (MCDSS). The key areas of GIS application in mining can be summarised as follows:

- xii. **Mineral exploration:** Identifying and mapping mineral resources using geological and geophysical data to guide exploration activities and target potential deposits.
- xiii. **Geological mapping:** Integrating and visualising geological information, such as rock

types, structures, and geological history, to better understand the geology of a mining area.

- xiv. **Geophysical mapping:** Analysing and displaying geophysical data to reveal subsurface properties and structures, such as anomalies, interfaces, and potential resource deposits.
- xv. **Spatial analysis:** Conducting spatial analyses to determine the relationships between geological, geophysical, and other spatial data for informed decision-making. Proximity of mines to critical ecosystems, which need to be protected, is easily carried out using GIS. Area calculations are executed with ease using GIS, based on a projected coordinate system, such as UTM.
- xvi. **Resource estimation:** Estimating the size, quality, and distribution of mineral resources in a mining area using spatial data and statistical modelling techniques.
- xvii. **Environmental assessment:** Evaluating the potential environmental impacts of mining activities by analysing spatial data related to land use, hydrology, and ecosystems.
- xviii. **Geohazard mitigation:** Identifying and monitoring geological hazards, such as landslides, subsidence, and seismic activity, to minimize risks to mining operations and surrounding areas.
- xix. **Mine planning and design:** Using GIS tools to design and optimise mining operations, considering factors such as access, infrastructure, waste disposal, and resource extraction.
- xx. **Infrastructure and logistics management:** Planning and monitoring the transportation, storage, and processing of extracted resources using spatial data and GIS tools.
- xxi. **Reclamation and closure:** Evaluating and planning the reclamation and closure of mining sites, ensuring the restoration of the environment and land use compatibility.

5.5 Policy, Legal and Regulatory Aspects in Mining Surveys

Mining rights are tightly linked to sensitive issues and questions around land tenure and the physical environment, hence to land and environmental rights as well. Many studies across the world have established extensive violations of human rights in the mining sector, a key example being the case of Taita Taveta, Kenya, as confirmed in 2016 through a public inquiry led by the Kenya National Commission on Human Rights (KNCHR). The inquiry found that the violations were widespread, from land rights, environmental rights, gender rights, children rights, to labour rights. Key to gaining a shared understanding of the extent of such violations of rights and resolving them is the geospatial exactitude obtainable from accurate mining surveys and mapping.

Conducive policies, laws, and regulations are needed to resolve and manage conflicts in the mining sector. Mining surveys provide the spatially explicit metrics and parameters required to accurately allocate, enforce, and monitor land-related mining rights. Maps generated using various techniques based on data from various sources e.g., aerial photogrammetry, laser scanning – both terrestrial laser scanning (TLS) and airborne laser scanning (ALS), satellite imagery, topographical surveys, cadastral surveys, and GIS complement the datasets and information needed for sound decision support when administering such crucial matters of land and mining rights, all the way to minimising the negative environmental effects of mining and ensuring adequate rehabilitation after mine closure.

The evolution of mining legislation has realised significant milestones, not least in Africa. Kenya, an East African economic giant, introduced in 2016 a modern mining law that has been referred to as Africa's most progressive mining law. The Mining Act

of 2016 specifies how mining rights should be administered based on a digital mining cadastre and units of blocks defined by graticules spaced 15" apart. This Kenyan example confirms the important role of mining surveys and mapping in managing and administering mining activities and mining rights.

The given illustration of a mining block serves as a suitable example on how to translate legal and policy aspects into practice when issuing mining rights or licenses attached to land parcels (Fig. 4.3). ABCD is the **pseudoquadrilateral** representing a mining block. In the Kenya Mining Act of 2016, for example, such a block is bounded by meridians spaced 15 arc seconds apart and parallels spaced 15 arc seconds apart. As shown based on a spherical Earth model, R is the radius of the Earth, estimated to be 6 371 km.

Generally:

- **Arc length for AB or CD = (longitude difference, i.e., $\lambda_2 - \lambda_1$ in degrees/360) * $2\pi(R\cos\Phi)$** , where Φ is the latitude (in degrees) of the circle of latitude, along which the longitude difference is measured. Near the equator (OE), $R\cos\Phi$ is almost equal to unity.
- **Arc length for AD or BC = (latitude difference, i.e., $\varphi_2 - \varphi_1$ in degrees/360) * $2\pi R$** , since all meridians are great circles such that R is the Earth radius always.

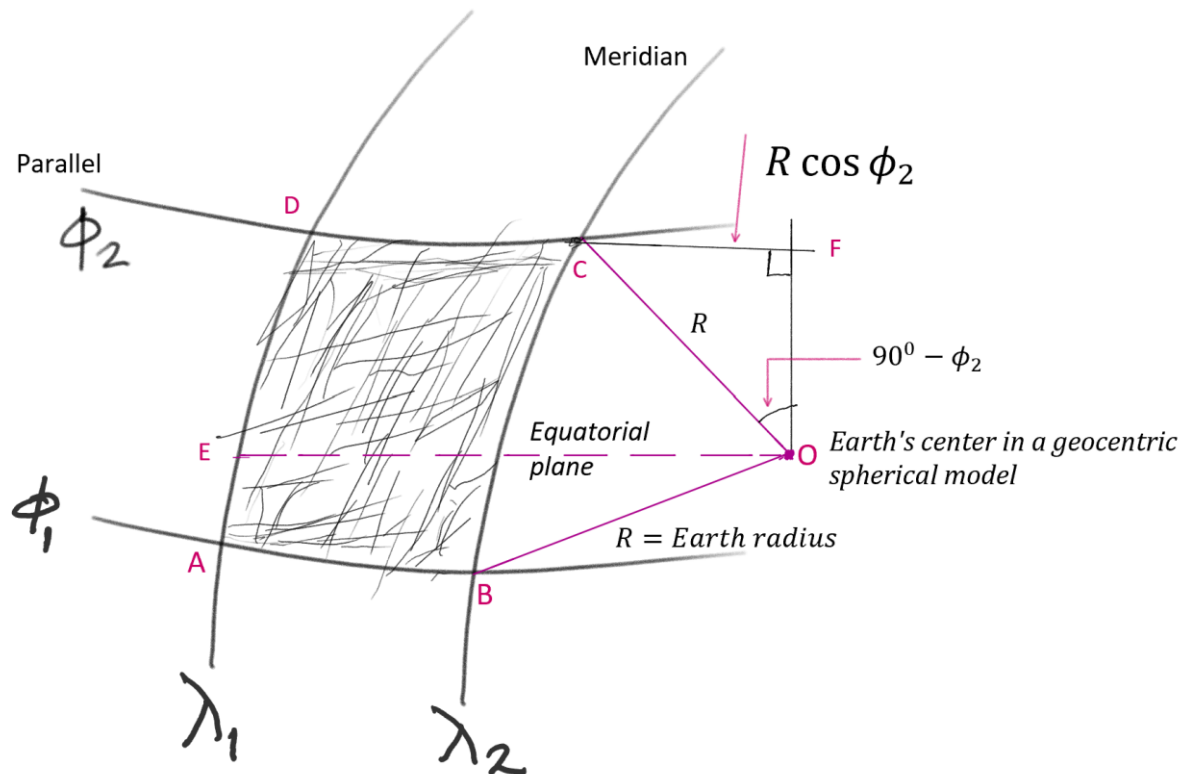


FIGURE 4.3

A pseudoquadrilateral ABCD representing a mining block based on a geocentric spherical model of the Earth

6. Bibliography

Adero, N.J. (2003). *The Role of Surveying in Tunnelling: Practical Experience*. Technical Report documenting professional experience as a Tunnel Surveyor, Sondu-Miriu Hydropower Project, Kenya, 2001 - 2003.

Adero, N.J., Drebenstedt C., Prokokofeva E.N., & Vostrikov A.V. (2020). Spatial data and technologies for geomonitoring of land use under aspects of mineral resource sector development. *Eurasian Mining* 2020. No. 1 (33), 69-74.

Beaumont, C., & Pincus, M. (2008). *Introduction to Mineral Exploration*. Oxford University Press.

Bishop, P., & Williams, P. A. (2008). *Geological maps: an introduction*. Geological Society, London, Special Publications, 294(1), 1-13.

Drebenstedt, C., & Singhal, R. (eds.), 2014. *Mine Planning and Equipment Selection*. Switzerland: Springer Int. Publishing.

Elsevier. Schlumberger Limited. (2017). *What is geophysical mapping?* Retrieved from <https://www.slb.com/services/geophysical/acquisition-processing/land-data-acquisition/what-is-geophysical-mapping.aspx>

GoK - Government of Kenya (2016). *The Mining Act (No. 12 of 2016)*. Kenya Gazette Supplement No. 71 (Acts No. 12). Nairobi: Government Printer.

Hong-lan, W. (1987). "Analysis of the motion of a gyro-theodolite", *Applied Mathematics and Mechanics*, 8: 889–900, doi:10.1007/BF02019527.

Klitzsch, E. (2009). *Geophysical exploration*. John Wiley & Sons.

KNCHR – Kenya National Commission on Human Rights (2017). *Public Inquiry Report on Mining and Impact on Human Rights: Taita Taveta County*, 2016. Nairobi: Top Quality Converters Co.

Mackay, A. (2013). *Introduction to geophysical prospecting*.

Miller, G. (2019). *The Pioneering Maps of Alexander von Humboldt*. *Smithsonian.com*, official journal published by the Smithsonian Institution in Washington, D.C. October 15, 2019 issue.

Ogaja, C., Adero, N., & Koome. D. (2023). *Project Design for Geomatics Engineers and Surveyors*, 2nd edition. UK: Taylor & Francis Group, CRC Press. Ready to order from [Routledge.com](https://www.routledge.com)

Rubey, W. W. (1959). *Geology in engineering practice*. John Wiley & Sons.

Schofield, W. & Breach, M. (2007). *Engineering surveying*. Butterworth-Heinemann, pp. 519–533, ISBN 0-7506-6949-7.

Spectra Geospatial (2022). Z-Blade. Product Details. <https://spectrageospatial.com/z-blade>

van Wegen, W. (2018). *Surveying in the Mining Sector: An Overview of Geospatial Methods in Mining Engineering*. GIM International, January 12, 2018. <https://www.gim-international.com/content/article/surveying-in-the-mining-sector>

Zielinski, S. (2010). *Cholera, John Snow and the Grand Experiment*. *Smithsonian.com*, official journal published by the Smithsonian Institution in Washington, D.C., August 18, 2010 issue.

7. Practice Exercises

Students should attempt the following practice-oriented questions then refer to the recommended solutions later.

7.1 GIS Application Exercise

1. Use GIS software of your choice to make a digital map of the Counties in Kenya to recommended cartographic standards. From a spreadsheet of population data for each county ranging from 1999 to 2019, show a mapped trend in population sizes and population density for any two counties of your choice over that period. Present the mapped data as a layout to be shared as a pdf file. Detail the procedures you have used.
2. Zoom in to Taita Taveta University, Kenya, from any digital map service like Google Earth. Sketch a mining block around it bound by 15 arc seconds, as provided for in the Kenya Mining Act of 2016. Prove that the linear dimensions of the sides of the block measure about 460 m. Based on this, deduce the number of mining blocks obtainable from an area of 800 ha.

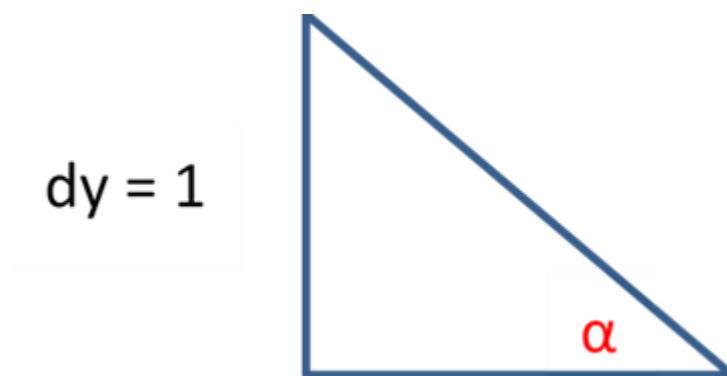
Practice Exercises

7.2 Engineering Application Exercise

3. A straight segment of a railway has a design gradient of **0.5 degrees**. You are required to survey from the downstream end at chainage **295 + 125** to the upstream end at chainage **295 + 140**. The upstream level should have a reduced level of **597.000** metres.
- (a) Find **X** if the plan drawing expresses the railway gradient as **1 in X**.
- (b) Calculate the correct staff reading at the downstream end if a backsight reading of **3.025** metres has been taken on a TBM whose reduced level is **594.945** metres.
- (c) List the survey **instruments, tools** and **accessories** that would be suitable for this exercise, grouping them by functions under horizontal control and vertical control.

Solution

(a)



Making dy equal to UNITY, gradient (slope) = $dy/dx = 1/X = \tan \alpha$, where $\alpha = 0.5^\circ$

$$X = 1/\tan 0.5^\circ = 114.59 \text{ or } 115$$

Expressed as **1 in 115**

(b) **dx** is calculated as the difference between the chainages = **15 m**

dy = gradient as a (decimal) fraction***dx** = $\tan\alpha$ ***dx** = **0.131m**

Downstream level = upstream level – **dy** = 597.000 – 0.131 = **596.869 m**

Height of Collimation (HI) = RL of X(BM) + staff reading at X(BM) = 594.945 + 3.025 = **597.970 m**

Staff reading downstream = HI – RL downstream = 597.970 – 596.869 = **1.101 m**

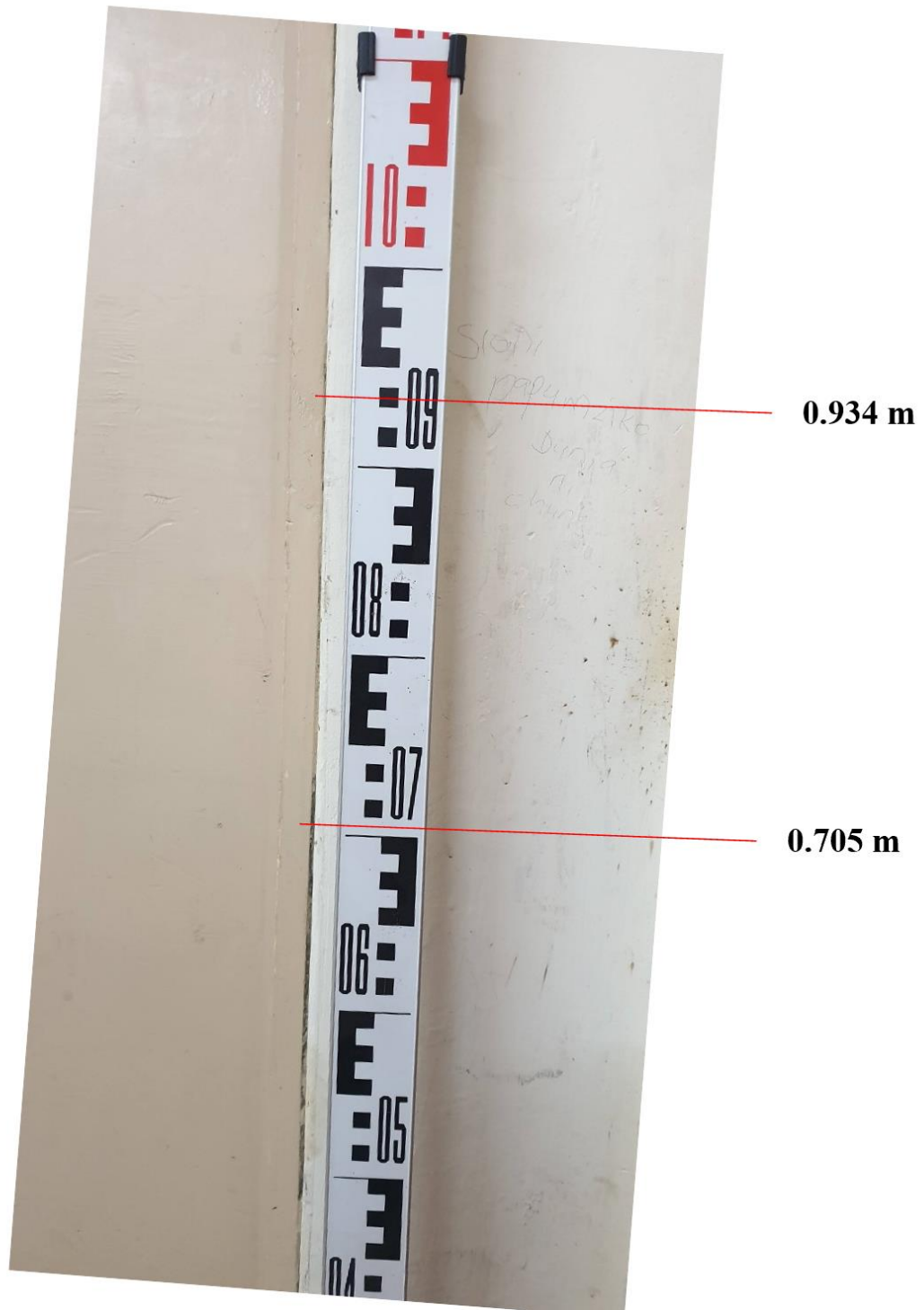
Check: Staff reading upstream = 597.970 – 597.00 = 0.970 m (hence higher than downstream by 1.101 – 0.970 = **0.131 m**...proved).

(c) Horizontal control: Instruments can be theodolite, tachymeter/total station, or geodetic GPS. Tools can be a tape measure, ranging rod/pole, or a string. Accessories include a tripod, reflector/prism, or any other visible target.

Vertical control: The most suitable instrument is a level. Tools may include a plumb bob, spirit bubble, tape measure and strings. Accessories include a levelling staff.

4. Make a simplified illustration on how to take readings off a levelling staff. A typical solution is shown below.

Practice Exercises



Levelling staff

(Photo credit: Nashon Adero, Taita Taveta University Survey Lab, 17-09-2022)

Note that a levelling staff is used together with a **level**. It is held vertically on the target point whose height is being measured. There are major graduations of **100 mm**, minor graduations of **10 mm** forming small blocks, and some of the **50 mm** blocks out of the major **100 mm** block are joined to form (inverted) **E** patterns.

5. A surveyor uses a Total Station whose accuracy is specified in the operator's manual as **10 mm + 10 ppm**. The instrument measures a chord length to be 150.030 m where the intersection angle is 60° for a curved tunnel of radius, $R = 150$ m.

Based on the information above:

- (i) Determine if this Total Station is fit for use on the site
- (ii) Advise the engineers on the action to take based on this finding

Solution

Suitability of the Total Station

- (i) Proof

$\delta = \text{half of intersection angle (IA)} = 30^\circ$

Theoretical chord length = $2R \sin \delta$

$$2(150 \text{ m}) \sin 30^\circ = 150.000 \text{ m}$$

Standard Deviation, SD, for theoretical chord length = $10 \text{ mm} + 10 \times (150.000/1000)$
 $\text{mm} = 11.500 \text{ mm}$

Rule:

Practice Exercises

The measurement is acceptable **if:** Observed difference $\leq 2SD$

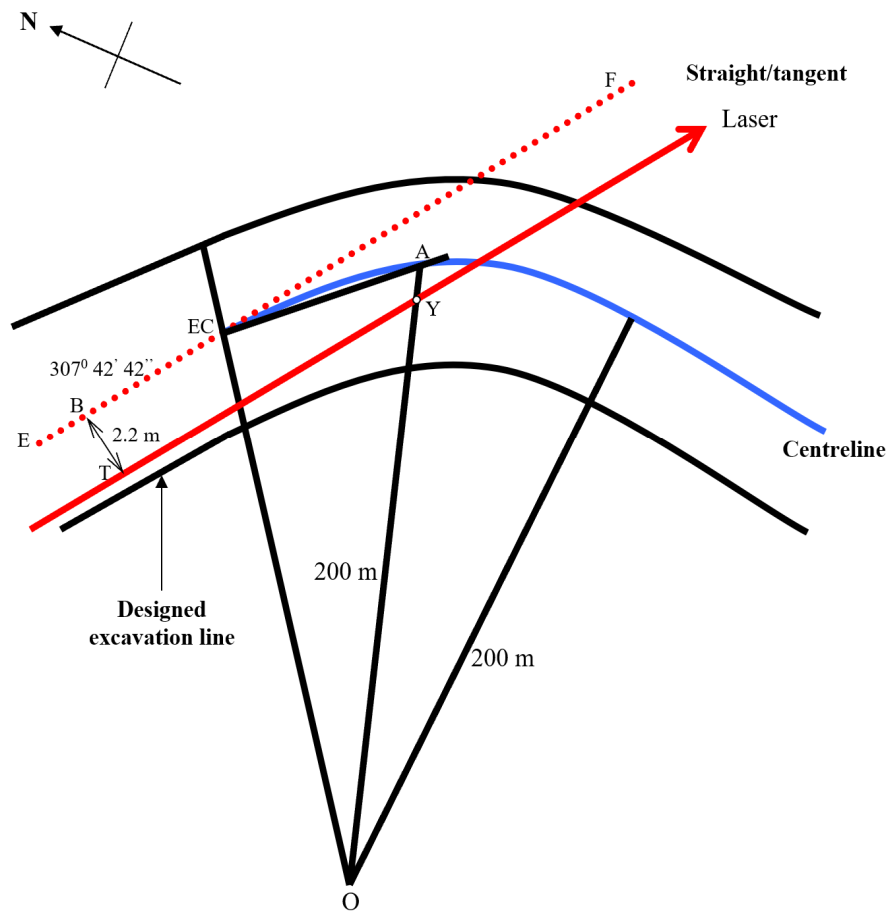
Observed difference = 150.030 m – 150.000 m = 0.030 m = **30 mm**

2SD = **23.000 mm**

Observed difference >2SD, hence not acceptable!

(ii) Advice: Replace the instrument or use the instrument after calibration and further verification of accuracy against a known distance.

6. Refer to the diagram below showing the geometry of one curved upstream section of a water tunnel. It is based on the Sondu-Miriu Hydropower project in Kenya. In the curved section of $R = 200$ m, the engineering surveyors used formwork spans measuring only three metres in length, a choice that can be mathematically proven given $R = 200$ m, because the curve length and chord length for such a short span do not vary significantly, but over several spans the errors can accumulate significantly if not checked. Solve for the offset distance AY to the designed centreline. (**Hint:** In order to get accurate results, the intermediate answers obtained should not be rounded off).



Solution

EF is the straight tangential to the curve at EC. This straight is also the centreline of the adjoining straight section. The laser beam is parallel to the straight EF at a separation distance of 2.2 metres. The centreline of the curved section is as shown. The coordinates of EC are given as (709 390.152 mE, 9 956 751.906 mN) and its chainage is 0+108.442 m. The chainage of point A, on the curve, is 0+109.482 m.

Task: To determine the offset from the laser beam to the centre of the tunnel at chainage 0+109.482 m, i.e., point A.

Practice Exercises

Distance from EC to A = chainage of A – chainage of EC = $109.482 - 108.442 = 1.040$
m (curve length)

Bearing of A from EC = $307^{\circ} 42' 42'' - 180^{\circ} + \delta$

δ is the tangential angle = $(90 \times \text{curve length})/(\pi R)$ degrees = $(90 \times 1.04)/200\pi$ degrees

\therefore Bearing EC \rightarrow A = $127^{\circ} 51' 38''.2$

Chord length = $2R\sin\delta = 1.040$ m

Coordinates of A

By polar to rectangular computation ($s = 1.040$ m; $\alpha = 127^{\circ} 51' 38''.2$)

$\Delta N = -0.638$ m $\Delta E = +0.821$ m

$N_A = 9\,956\,751.268$ m $E_A = 709\,390.973$ m

Coordinates of O (centre of curve)

EC \rightarrow O

By polar to rectangular computation ($s = 200$ m; $\alpha = 217^{\circ} 42' 42''$)

$\Delta N = -158.220$ m $\Delta E = -122.338$ m

$N_O = 9\,956\,593.686$ m $E_O = 709\,267.814$ m

Determining equation of the radius A \rightarrow O

Using coordinates of A and O

$$\text{Gradient (m)} = \Delta E / \Delta N = 0.782$$

$E = mN + c$ (equation of a straight line in rectangular coordinates)

$$\therefore c = E - mN = -7\,072\,363.763$$

$$E = 0.782N - 7\,072\,363.763 \text{-----(i)}$$

Determining equation of the laser beam

m = gradient of the tangent EF (parallel to the beam)

Taking any other point on the line EF, say B at 100 m from EC, so that

$$s = 100 \text{ m and } \alpha = 307^{\circ} 42' 42''$$

By polar to rectangular computation ($s = 100 \text{ m}$ and $\alpha = 307^{\circ} 42' 42''$)

$$\Delta N = +61.169 \text{ m}$$

$$\Delta E = -79.110 \text{ m}$$

$$m = \Delta E / \Delta N = -1.293$$

Coordinates of T (2.2 m perpendicular to B)

By polar to rectangular computation ($s = 2.2 \text{ m}$; $\alpha = 217^{\circ} 42' 42''$)

$$\Delta N = -1.740 \text{ m}$$

$$\Delta E = -1.346 \text{ m}$$

$$N_T = N_{EC} + 61.169 - 1.740 = 9\,956\,811.334 \text{ m}$$

Practice Exercises

$$E_T = E_{EC} - 79.110 - 1.346 = 709\,309.696 \text{ m}$$

$$c = E - mN = 13\,586\,498.196$$

$$E = -1.293N + 13\,586\,498.196 \text{-----(ii)}$$

Solving the two equations (i) and (ii) simultaneously, coordinates of the intersection point Y are obtained as:

$$N_Y = 9\,956\,749.536 \text{ m}$$

$$E_Y = 709\,389.620 \text{ m}$$

Now, using the coordinates of A and Y, by rectangular to polar computation, $Y \rightarrow A$ becomes $37^\circ 59' 46'' @ 2.197 \text{ m}$.

$$\therefore \text{Offset } YA = 2.197 \sin (\alpha \text{ laser beam} - \alpha \text{ YA}) = 2.197 \text{ m}$$

N/B: Because the figures presented have been rounded off, some of them look the same though they are different. The surveyor must take this fact seriously and use the figures as computed and stored in a calculator or computer so as to avoid the accumulation of systematic errors in such sensitive tunnel surveys. Regular checks after several metres are necessary during construction in order to contain such errors.

7. When assessing the relationship between angular and linear precisions of optical survey instruments, the sighting distance is critical. Illustrate mathematically how the angular and linear dimensions relate. Hence, show how the following relationships between the angular precision of an instrument and the linear tolerance come about.

(Hint: Consider how the arc length of a circle (linear tolerance) is mathematically related to the angle it subtends at the centre for a given radius, r).

- i. 20" is equivalent to 10 mm at a sighting distance of 100 m
- ii. 10" is equivalent to 5 mm at a sighting distance of 100 m
- iii. 5" is equivalent to 2.5 mm at a sighting distance of 100 m
- iv. 1" is equivalent to 0.5 mm at a sighting distance of 100 m

Also show that if 5 mm tolerance has been specified for site work up to a distance of 100 m, a 10" theodolite (or total station) would be suitable. By simple proportion, show that if 5 mm tolerance has been specified but the maximum distance to be set out is 50 m, a 20" instrument is sufficient.

8. For a straight 60-metre-long section of the Northern Collector Tunnel from Murang'a to Thika in Kenya, a design gradient of 1 in 55 has been specified. The downstream chainage is 0+095 and the chainage values are increasing upstream.
- a) Determine the chainage for the upstream end of this section
 - b) If a surveyor has observed a backsight of **1.509 m** at the Benchmark whose Reduced Level is **1,799.162 m** and given that the Benchmark is **0.582 m** higher than the invert level at the downstream end of this section, determine:
 - i. The invert level of the upstream end
 - ii. The staff reading at the upstream end from the same instrument station
 - c) If the tunnel is to be concrete-lined using a **15-metre**-long formwork/shutter beginning from the chainage **0+095**, calculate the chainage and the correct staff reading from the same instrument station expected at the downstream invert level of the third (3rd) formwork/shutter from this chainage.

Solution

- a) 60 m + 95 m = 155 m, hence **0+155**

Practice Exercises

- b) (i) Reduced Level of downstream end = $1799.162\text{m} - 0.582\text{m} = \mathbf{1798.580\text{ m}}$
 $dy/dx = 1/55$, but $dx = 60\text{m}$ for upstream end
 $\mathbf{dy} = 60\text{ m}/55 = 1.091\text{m}$ (height difference from downstream)
Upstream level = $1.091\text{ m} + 1798.580\text{ m} = 1799.671\text{ m}$
- (ii) Height of Collimation = RL at BM + BS = $1799.162 + 1.509 = 1800.671\text{ m}$
Staff reading upstream = Height of Collimation – RL upstream
Height of Collimation = $1800.671\text{ m} - 1799.671\text{ m} = \mathbf{1.000\text{ m}}$
- c) Due to continuity, downstream position of the 3rd shutter = upstream position of the 2nd shutter = **30 m slope distance** from the starting point.
Horizontal Equivalent (dx) = $30 \cos((\tan^{-1}(1/55)) = \mathbf{29.995\text{ m}}$.
Chainage = $0+095 + 29.995\text{ m} = 0+124.995$ (rounded off to **0+125**)
 $\mathbf{dy} = 30 \sin(\tan^{-1}(1/55)) = \mathbf{0.545\text{ m}}$
Staff reading downstream of 3rd shutter = Staff reading at chainage 0+095, i.e.,
 $(1800.671 - 1798.580) - 0.545 = \mathbf{1.546\text{ m}}$

9. A tunnel section of the Sondu-Miriu Hydropower Project has a designed slope of 1 in 300.5. The chainage 0+000m is reckoned to be at the junction of Adit 1 and the main tunnel leading from intake facilities to the power station. Blinding levels are to be indicated at 2 m intervals upstream from chainage 0+100 m to 0+110 m. The designed invert level of the final concrete lining at chainage 0+000 is 1386.208 m, and the level of blinding is designed to be 300 mm below the invert level. Using this information, compute the data for setting out the levels. Assume that a level has been set up so that the backsight to a TBM whose reduced level (RL) is 1387.200 m reads 0.301 m.

Solution

$$\begin{aligned}\text{Height of collimation (HI)} &= 1387.200 + 0.301 = 1387.501 \text{ m} \\ \text{Blinding level at 0+000} &= 1386.208 - 0.300 = 1385.908 \text{ m} \\ \text{dy} &= \text{slope} \times \text{distance} \\ \therefore \text{blinding level at 0+100 m} &= 1385.908 + 100/300.5 \\ &= 1386.241 \text{ m}\end{aligned}$$

$$\text{Grade staff reading at 0+100 m} = \text{HI} - 1386.241 = 1.260 \text{ m}$$

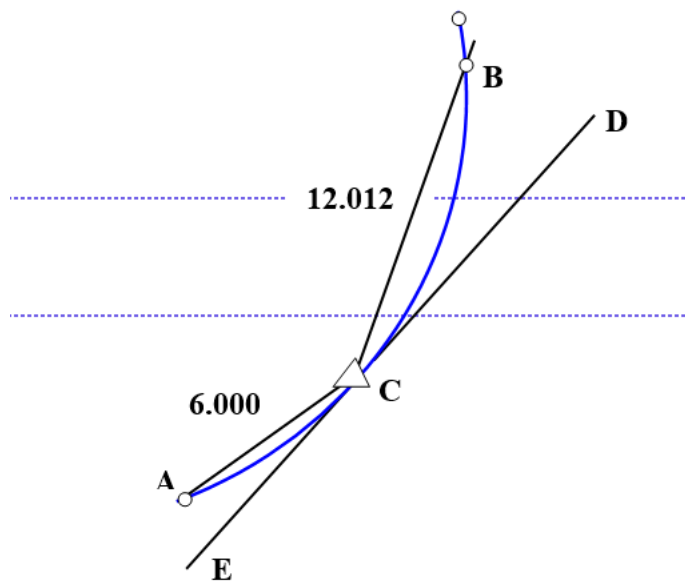
Calculating the rest similarly gives the following setting out data for blinding levels:

Chainage (m)	Blinding level (m)	HI (m)	Grade staff reading (m)
0+000	1385.908	1387.501	
0+100	1386.241		1.260
0+102	1386.247		1.254
0+104	1386.254		1.247
0+106	1386.261		1.240
0+108	1386.267		1.234
0+110	1386.274		1.227

10. A curved tunnel section is to be constructed to have a gradient of 1 in 300.5 and interior diameter of 4.200 m. The radius of the curve is 200 m. Concrete lining is to be placed at 3 m intervals with the help of a lining shutter of length 3 m. The shutter is positioned so that its downstream end is at the beginning of the curve, A, chainage 0+260 m. The spring level at chainage 0+000 m in the adjoining straight section is designed to be 1377.769 m. A total station is set up at 12.012 m from the upstream end, B, of the shutter and 6.000 m from point A along the centreline. A level machine has been set up to attain a height of collimation (HI) = 1376.833 m. As the surveyor in charge of this construction, you are required to perform all the essential measurements to set out the vertical alignment using readings off an inverted tape and related horizontal alignment too. Show your work.

Solution

Practice Exercises



As shown in the sketch above, the upstream end of the shutter is at point B. ED is tangential to the curve at the instrument station, C. The curve defines the centreline of the curved tunnel.

Vertical alignment

Chainage of upstream end is 0+263 m (curve length practically equal to chord length for 3 m interval given $R = 200$ m)

$$dy = 1/300.5 \times 263 \text{ m}$$

$$\begin{aligned} \text{Spring level at 0+263m} &= 1377.769 + dy \\ &= 1378.644 \text{ m} \end{aligned}$$

$$\text{Zenith level upstream} = 1378.644 + 2.1 = 1380.744 \text{ m.}$$

$$\begin{aligned} \text{Similarly, zenith level downstream} &= 1/300.5 \times 260 + 1377.769 + 2.1 \\ &= 1380.734 \text{ m} \end{aligned}$$

After balancing the sides, the tape readings for setting out the correct zenith/summit levels upstream and downstream are:

$$\text{Downstream: } 1376.833 - 1380.734 = - \mathbf{3.901m}$$

$$\text{Upstream: } 1376.833 - 1380.744 = - \mathbf{3.911m}$$

Using adjustment screws, the zenith levels downstream and upstream are adjusted until the inverted tape readings above are obtained. The diameter of 4.200 m is then set out along the vertical axis and horizontal axis to achieve a perfect circular shape.

Horizontal alignment

Calculating the tangential angles ACE and BCD,

$$\text{Angle ACE} = \sin^{-1} (\text{chord length}/2R) = \sin^{-1} (6.000/2R) = 00^{\circ} 51' 34''$$

$$\text{Angle BCD} = \sin^{-1} (\text{chord length}/2R) = \sin^{-1} (12.012/2R) = 01^{\circ} 43' 15''$$

$$\text{Angle ACB (Obtuse)} = \mathbf{177^{\circ} 25' 10''}$$

If the horizontal angle reading to A is set to $00^{\circ} 00' 00''$, then the horizontal angle reading to be set when aligning the upstream end is $\mathbf{177^{\circ} 25' 10''}$. The lining shutter is then adjusted sideways until the tip of a plumb bob hung from its zenith at this end coincides with the cross hairs at this reading.

$$\text{Curve length CB} = \delta \pi R / 90 = 12.014 \text{ m.}$$

Now, considering that a chord length of 3 m equals to a curve length of 3 m to the millimetre accuracy required in this case with $R = 200 \text{ m}$, the curve length from C to the required position of downstream end of the 3-metre-long shutter is 15.014 m.

$$\text{Given curve length} = 15.014 \text{ m}$$

$$\text{Then corresponding chord length} = 2R \sin \delta$$

$$= 400 \sin[(90 \times 15.014) / \pi R] = 15.010 \text{ m}$$

Thus, the reading for aligning the downstream end becomes:

$$180^{\circ} - \sin^{-1} (6.000/400) - \sin^{-1} (15.010/400) = \mathbf{176^{\circ} 59' 24''}$$